

# The prediction of effective thermal conductivities perpendicular to the fibres of wood using a fractal model and an improved transient measurement technique

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Received 31 August 2005; received in revised form 20 December 2005

Available online 12 June 2006

## Abstract

The porous microstructure of wood samples on their sections perpendicular to the fibres were analyzed using the scanning electron microscope images. The fractal dimensions of these images were calculated using the box-counting method, respectively. They are all approximately equal to 1.4, although the distribution and the scale of wood fibres are extremely different. Then, a fractal model for predicting the effective thermal conductivities of wood was established using the thermal resistance method. In addition, we measured the effective thermal conductivity of wood via an improved transient plane source measurement method. The calculated results by the proposed model are in good agreement with the experimental data as well as the literature data. The comparison shows clearly that this fractal model can be used to accurately and effectively predict the effective thermal conductivities perpendicular to the fibres of wood. © 2006 Elsevier Ltd. All rights reserved.

**Keywords:** Wood; Fractal dimension; Fractal model; Microstructure of wood; Tangential thermal conductivity; Radial thermal conductivity; Transient measurement

## 1. Introduction

Wood is a typical material used in architecture and decoration, as well as a widely used solid fuel. Thermal properties of wood are needed in applications and the effective thermal conductivity is the most important one for modeling the heat conduction, combustion and pyrogenation process of wood. Recently, Thunman et al. [1] reported their study of models for calculating the thermal conductivity of wood at different stages of combustion. Using these models, they calculated the thermal conductivity perpendicular to the fibres and the thermal conductivity along the fibres of wood. Asako et al. [2] studied the thermal conductivity of compressed woods by a three-dimensional hot-wire method. As previously shown in these references, the

analysis of the microstructure of wood fibres is an effective way to study the heat conduction of wood.

Fractal is a word firstly coined by Mandelbrot [3], then it has been widely used in so many fields, especially in the non-linear and micro-scale science. The fractal theory can well describe the disorder and stochastic performance of porous media. Pitchumani et al. [4] have firstly used fractal theory in the research of the effective thermal conductivity for unidirectional fibrous composites. Then, it also has been successfully used in the study of the prediction of effective thermal conductivities of bidispersed porous media [5] and liquid with nanoparticles [6], but few reports used this theory to study the effective thermal conductivity of wood.

When it comes to measure the thermal properties of solid materials, the transient plane source (TPS) method is widely used [7–10]. This transient measurement technique also appropriately satisfies the requirements for the measurement of the thermal properties of wood. Therefore,

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## Nomenclature

$a$	scale of fractal units
$A$	area of a cell
$b$	thickness of a cell wall
$d$	fractal dimension
$h$	height of cavity of a cell
$k$	thermal conductivity
$l$	width of a cell
$q$	heat flux
$R$	thermal resistance
$t$	time
$T$	temperature

### Greek symbols

$\alpha$	thermal diffusivity
$\delta$	thickness of wood sample
$\phi$	porosity

### Subscripts

w	wall
c	cavity
t	tangential
r	radial

we use the TPS method in this paper, but it has been redesigned by us.

The major purpose of this paper is to establish a fractal model for calculating the effective thermal conductivity of wood perpendicular to fibres. First, the characters of scanning electron microscope (SEM) images of several wood samples are analyzed using the fractal theory. Then the fractal model is established via the thermal resistance method. Secondly, an improved TPS method is introduced and then performed to measure the effective thermal conductivity perpendicular to fibres of two kinds of wood samples. Finally, the results calculated by the fractal model are compared with the experimental data and the available literature data.

## 2. Microstructure of wood

### 2.1. Experimental observation

We select four kinds of widely seen and used woods the observed samples, including basswood, birch, larch and Korean pine. Although the structures of wood fibres differ in size, the shape and the distribution of them have certain similarities. In view of this fact the scanning electron microscope PHILIPS-XL30ESEM is used to clearly observe the microstructure of these samples. They are made by the central parts perpendicular to the fibres of wood, which were taken from the north-east and south-east forests of China.

The SEM images with different amplificatory multiples are displayed in Fig. 1. In the pictures (a) and (b), it is seen obviously that the boundary of fibres both in basswood and birch is very complex and the fibres distribute randomly. However, the pictures (c) and (d) of larch and Korean pine are extremely different from the pictures of basswood and birch. The pores of larch are very like a course of bricks, which are built fitly, and the array of the pores of Korean pine is very like a honeycomb. This means that the fibre distribution of softwood is more regular than that of hardwood, and there are certain self-similarities of wood fibre, either softwood or hardwood. The traditional Eculidean geometry cannot be used to describe

the distribution character of these microstructures in detail because of the regular dimension cannot represent the complexity of the images. According to the characters of fractal [3], these microstructures of wood are randomly quasi-regular, and there is a self-similarity at certain scale. Therefore, they can be considered as fractal structures and the fractal theory can be used to study them.

### 2.2. Fractal dimension

Before using the fractal theory, it is necessary to introduce some definitions of fractal indexes. The fractal dimension  $d$  is the most important parameter for describing the fractals. A two-dimensional object, such as the SEM figures of wood we obtained previously, can be divided into  $N(a)$  self-similar smaller squares each of which is scaled down by the length  $a$  of the side. Therefore, the fractal dimension  $d$  can be defined as

$$d = \log N(a) / \log(1/a). \quad (1)$$

The fractal dimension, unlike the normal Euclidean dimension, need not be an integer and is often not an integer. Using this definition, which is so-called the box-counting method, we can calculate the dimensions of these SEM images. The fractal dimensions of these four kinds of wood are calculated as shown in Fig. 2, respectively. The slopes of these four lines are the fractal dimensions, and they are listed in Table 1.

It should be noted that the fractal dimension of wood fibres cannot be accurately computed. One reason is that the uncertainty of the figure division through a certain figure threshold, another reason involves that the anisotropy of wood. Additionally, due to the diversity between different kinds of wood, there will be different fractal dimensions of wood in different directions for the same samples and for the different samples made from the same tree. However, from the results displayed in Table 1, it is to be aware that the fractal dimensions of wood fibres in the cross-sections are roughly equal to 1.4, and we can use the calculated fractal dimension from a certain sample to represent the fractal dimension of this kind of wood.

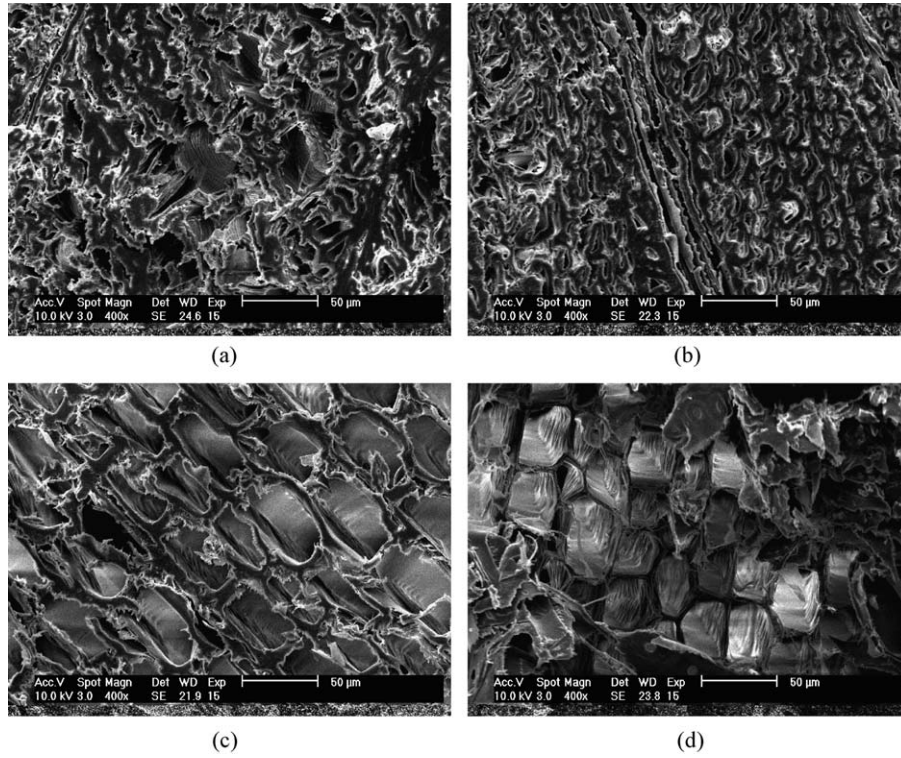


Fig. 1. SEM images of wood samples on their cross-sections: (a) basswood, (b) birch, (c) larch and (d) Korean pine.

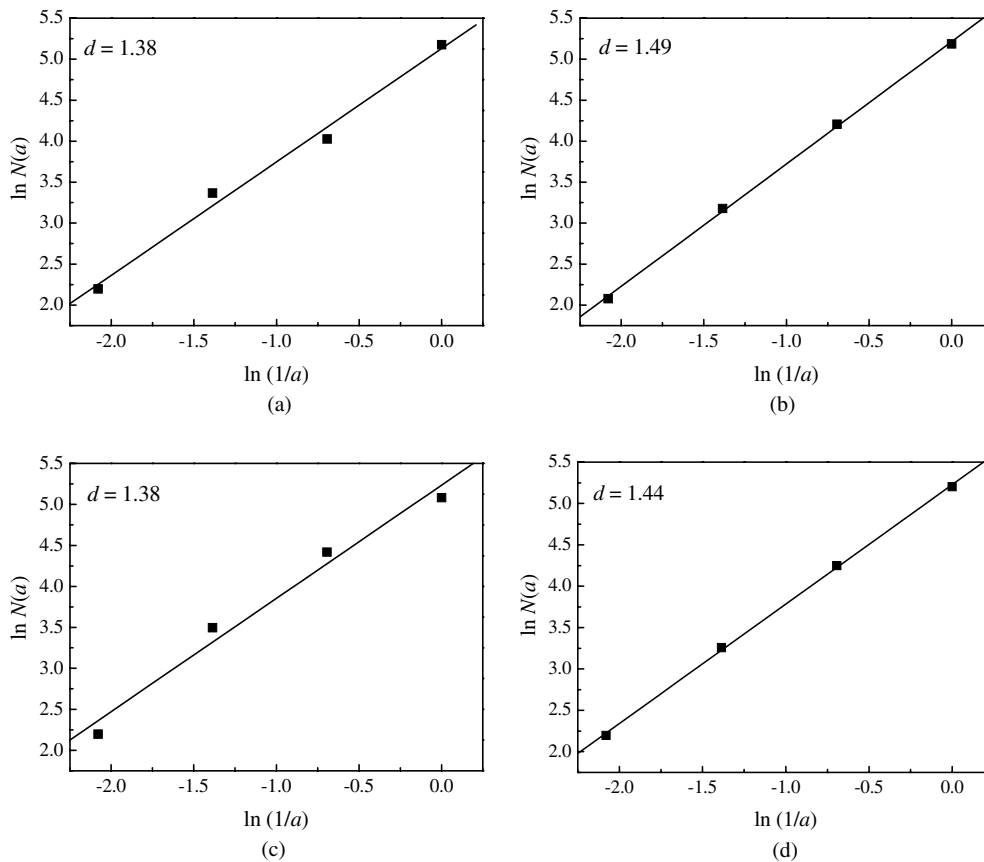


Fig. 2. Box-counting fractal dimensions of wood samples: (a) basswood, (b) birch, (c) larch and (d) Korean pine.

Table 1  
Box-counting dimensions of the SEM images of wood samples

Wood sample	Fractal dimension $d$
Basswood	1.38
Birch	1.49
Larch	1.38
Korean pine	1.44

### 3. Fractal model

As used in the references [1,2,5,11], thermal resistance simulation is a very useful method to simplify the heat conduction process in porous media and to predict the effective thermal conductivity. We also try to use this method in this paper to establish the relation between the average porosity and the effective thermal conductivities of wood. First, of all, several assumptions are made as follow:

- (1) The wood is dry, so there is no moisture in the cavity of cells.
- (2) The cell geometry is rectangular, and all cells have the same shape and dimensions.
- (3) The convective and radiative effects in the cavity of cells were neglected, and the heat transfer process of a cell is simplified to a pure heat conduction process.
- (4) The thickness of the cell wall is constant and is equal to  $b$ , the width of the cell is  $l$ , and the height of the cell cavity is  $h$  (as displayed in Fig. 3).
- (5) The direction  $x$  is set as the tangential direction, and the direction  $y$  as the radial direction.
- (6)  $k_w$  means the thermal conductivity of the cell wall of wood, and  $k_c$  means the thermal conductivity of the cavity of cells.

Then, the thermal conductivity of a wood cell can be used to represent the effective thermal conductivity of wood. The model of two-dimensional heat conduction in a wood cell and its separation are considered as shown in Fig. 3. Accordingly, the formulas of the effective thermal conductivities of wood can be obtained.

#### 3.1. Tangential thermal conductivity

When the direction of the heat flux is parallel to the  $x$  direction, the thermal resistance network of the tangential

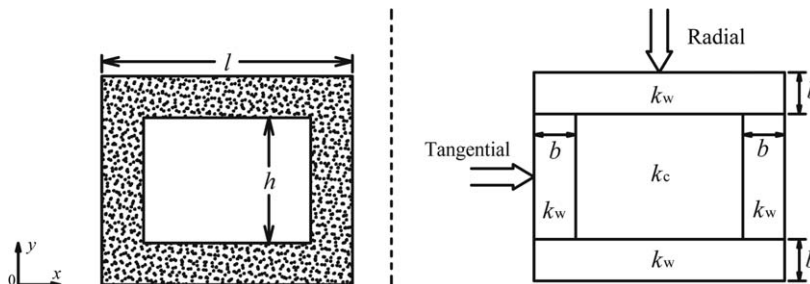


Fig. 3. Schematic diagram of a cell and its division.

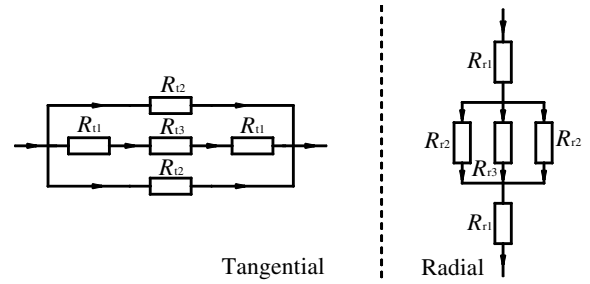


Fig. 4. Paths of heat flux through a cell represented by electrical circuits.

heat conduction can be described as represented in Fig. 4. If, moreover, we assume that the thickness of the cell at the direction perpendicular to this paper is the unit length 1, the thermal resistance of every part of the cell can be given as

$$R_{t1} = \frac{l}{bk_w}, \quad R_{t2} = \frac{b}{hk_w}, \quad R_{t3} = \frac{l-2b}{hk_c}, \quad (2)$$

and the whole thermal resistance of the cell is

$$R_t = \frac{l}{(h+2b)k_t}, \quad (3)$$

where  $k_t$  is the tangential effective thermal conductivity of wood. According to the paths of the thermal resistance network, the whole thermal resistance also can be written as

$$R_t = \left( \frac{2}{R_{t1}} + \frac{1}{2R_{t2} + R_{t3}} \right)^{-1}. \quad (4)$$

Substituting Eqs. (2) and (3) into Eq. (4), then it can be simplified to

$$k_t = \frac{(4b+hl)k_wk_c + 2b(l-2b)k_w^2}{2b(h+2b)k_c + (h+2b)(l-2b)k_w}. \quad (5)$$

#### 3.2. Radial thermal conductivity

When the direction of the heat flux is parallel to the  $y$  direction, the thermal resistance network of the radial heat conduction can be described as shown in Fig. 4. Therefore, the thermal resistance of every part of the cell can be given as

$$R_{r1} = \frac{b}{lk_w}, \quad R_{r2} = \frac{h}{bk_w}, \quad R_{r3} = \frac{h}{(l-2b)k_c}, \quad (6)$$

and the whole thermal resistance of the cell is

$$R_r = \frac{h + 2b}{lk_r}, \quad (7)$$

where  $k_r$  is the radial effective thermal conductivity of wood.

Like the above discussion of tangential effective thermal conductivity, according to the connection of the thermal resistances network as displayed in Fig. 4, the whole thermal resistance also can be given as

$$R_r = 2R_{r1} + \left( \frac{2}{R_{r2}} + \frac{1}{R_{r3}} \right)^{-1}. \quad (8)$$

Fitting Eqs. (6) and (7) into Eq. (8), then it can be rewritten as

$$k_r = \frac{2(h + 2b)(l - 2b)k_w k_c + b(h + 2b)k_w^2}{4b(l - 2b)k_c + (2b^2 + hl)k_w}. \quad (9)$$

### 3.3. Proposed model

On the one hand, in accordance with the previous six assumptions, the porosity  $\phi$  of wood can be given as

$$\phi = A_c/A, \quad (10)$$

where  $A_c$  is the cavity area of the cell, and  $A$  is the area of the cell. The expressions of  $A_c$  and  $A$  can be defined as

$$A_c = hl - 2bh, \quad (11)$$

$$A = hl + 2bl. \quad (12)$$

Substituting the Eqs. (10)–(12) into Eqs. (5) and (9), the correlation between the thermal conductivities and the porosity  $\phi$  can be obtained

$$k_t = \frac{0.5(1 + \phi)^2 k_w k_c + \phi(1 - \phi)k_w^2}{\phi(1 - \phi)k_c + 2\phi k_w}, \quad (13)$$

$$k_r = \frac{4\phi k_w k_c + 0.5(1 - \phi)k_w^2}{2\phi(1 - \phi)k_c + 0.5(1 + \phi)^2 k_w}. \quad (14)$$

According to another definition of fractal dimension [12], the relationship between the porosity and the fractal dimension also can be presented as

$$\phi = CA^{d-1}, \quad (15)$$

where  $C$  is a constant.

On the other hand, wood is a porous material and the porosity is dependent on the kind of wood as well as the moisture content. This porosity  $\phi$  was estimated by Suleiman et al. [9] using

$$\phi = 1 - (\rho_{ave}/\rho_{th}), \quad (16)$$

where  $\rho_{ave}$  is the average apparent density of the sample,  $\rho_{th}$  is the assumed theoretical density of a compact solid free from voids. The value of  $\rho_{th}$  is assumed to be 1500 kg/m<sup>3</sup> [9]. Table 2 gives the density and calculated porosity of wood samples, including both larch, Korean pine we measured and birch reported in reference [9]. The porosity

Table 2  
Density and calculated porosity of wood samples

Wood sample	Density (kg m <sup>-3</sup> )	Porosity
Larch	671	0.552
Korean pine	431	0.712
Birch I [9]	680	0.547
Birch II [9]	567	0.622
Birch III [9]	543	0.636

is defined by Eq. (16) using the corresponding listed densities. Moreover, as shown in Fig. 1, it is apparently seen that the scale of different kinds of wood fibres is in the range from 2 to 5  $\mu$ m. In order to build an all-purposed model for each kind of wood, we use the average scale that we have investigated previously. Then, it is assumed that  $A$  is  $4 \times 4 \mu$ m, and  $A_c$  is  $3 \times 3 \mu$ m. This also means that the porosity is about 0.56, which well accords with the average value of the listed porosities in Table 2.

Substituting this value into Eq. (15) and combining with the average fractal dimension which is approximately equal to 1.42, we can obtain that the value of constant  $C$  is about 1.73, then, Eq. (15) can be rewritten as

$$\phi = 1.73 \times 16^{d-1}. \quad (17)$$

Finally, after Eq. (17) has been substituted into Eqs. (13) and (14), the formulas that describe the relationship between the fractal dimension and effective thermal conductivities of wood are obtained

$$k_t = \frac{0.5(1 + 1.73 \times 16^{d-1})^2 k_w k_c + 1.73 \times 16^{d-1}(1 - 1.73 \times 16^{d-1})k_w^2}{1.73 \times 16^{d-1}(1 - 1.73 \times 16^{d-1})k_c + 3.46 \times 16^{d-1}k_w}, \quad (18)$$

$$k_r = \frac{6.92 \times 16^{d-1}k_w k_c + 0.5(1 - 1.73 \times 16^{d-1})k_w^2}{3.46 \times 16^{d-1}(1 - 1.73 \times 16^{d-1})k_c + 0.5(1 + 1.73 \times 16^{d-1})^2 k_w}. \quad (19)$$

## 4. Transient measurement

### 4.1. Measurement model

A new short time formula will be proposed in this paper to satisfy measurement requirements of the thermal properties of wet wood. The physical model of the wood sample we measured is displayed in Fig. 5. If the length and width of the measured samples are extremely longer than the thickness, the sample can be considered as a one-dimensional quasi-infinite slab. There is only a temperature gradient at the  $x$  direction through the sample, so the heat conduction process can be simplified to a one-dimensional process. When there is an even plane source with constant heat flux  $q$  under the sample to heat it, and the other surfaces of the sample are adiabatic, the differential equations that describe the one-dimensional heat conduction of the model can be given as

$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}, \quad (20)$$

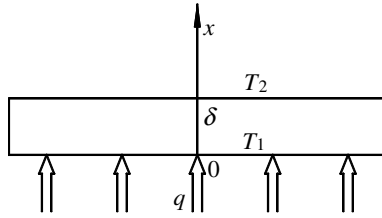


Fig. 5. Physical model for transient measurement.

subject to the boundary conditions

$$-k \frac{\partial T}{\partial x} = q, \quad \text{at } x = 0, \quad (21)$$

$$\frac{\partial T}{\partial x} = 0, \quad \text{at } x = \delta, \quad (22)$$

and the initial condition

$$T = T_0, \quad \text{for } t = 0, \quad (23)$$

where  $\alpha$  is the thermal diffusivity of the wood sample.

Taking the Laplace transformation to the equations from (20)–(23), the temperature variation in the sample parallel to  $x$  direction at every time  $t > 0$  can be obtained

$$T = T_0 + \frac{2q}{k} \sqrt{\frac{\alpha t}{\pi}} \sum_{n=0}^{\infty} \left[ \exp\left(-\frac{K_{n1}^2}{4t}\right) + \exp\left(-\frac{K_{n2}^2}{4t}\right) \right] - \frac{q}{k} \sqrt{\alpha} \sum_{n=0}^{\infty} \left[ K_{n1} \operatorname{erf}\left(\frac{K_{n1}}{2\sqrt{t}}\right) + K_{n2} \operatorname{erf}\left(\frac{K_{n2}}{2\sqrt{t}}\right) \right], \quad (24)$$

where  $n$  is zero or a positive integer, and

$$K_{n1} = \frac{2n\delta + x}{\sqrt{\alpha}}, \quad K_{n2} = \frac{2(n+1)\delta - x}{\sqrt{\alpha}}, \quad (25)$$

If a certain accuracy requirement is taken into consideration, the shorter the measurement time is, the fewer the items of Eq. (24) are required. In fact, we consider only a very short time after heating in the transient measurement process, so the items that  $n \geq 1$  can be deleted then Eq. (24) can be rewritten as

$$T = T_0 + \frac{2q}{k} \sqrt{\frac{\alpha t}{\pi}} \left\{ \exp\left(-\frac{x^2}{4\alpha t}\right) + \exp\left[-\frac{(2\delta - x)^2}{4\alpha t}\right] \right\} - \frac{q}{k} \left[ x \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) + (2\delta - x) \operatorname{erf}\left(\frac{2\delta - x}{2\sqrt{\alpha t}}\right) \right], \quad (26)$$

We called Eq. (26) the short time formula for the transient measurement.

#### 4.2. Experimental equipments

In the experiment, the larch and Korean pine are selected measurement samples. To satisfy the experimental requirements, the samples were produced from the original wood without cracks, notches and impurities. In addition, every kind of sample is produced four same specimens, whose sizes are all 100 mm × 100 mm × 15 mm, to accord with the assumptions of one-dimensional heat conduction.

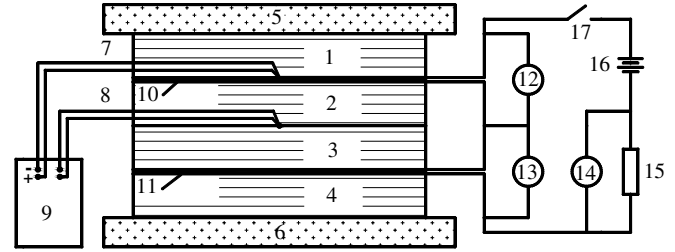


Fig. 6. Schematic diagram of measuring system. (1, 2, 3, 4) wood sample; (5, 6) insulator; (7, 8) thermal couple; (9) data collection chip; (10, 11) plane heater; (12, 13, 14) voltage meter; (15) precise resistance; (16) power source; (17) switch.

Their surfaces are processed extremely flat and smooth to the same testing conduction.

As shown in Fig. 6, the experimental equipment system consists of three main parts, namely: shelves for fixing the samples, digital devices for automatically collecting the experimental data, and circuits for heating. Concretely, data collection device mainly consists of several copper-nickel thermal couples, a HP 34970 A data collection chip, and a PC with some interfaces, and the heating circuit mainly consists of a foursquare plane heater, 220 V power source, a precise electric resistance, and so on.

The heat flux generated by the higher plane heater is equal to that by the lower one, and these two heaters are the same as each other, so the plane between the sample 2 and sample 3 can be considered as an adiabatic interface. Therefore, the condition of sample 2 and sample 3 well accords with the previous measurement model and the proposed short time formula can be used here. One of the thermal couples was fixed at the plane between sample 1 and sample 2, and another one was fixed at the plane between sample 2 and sample 3 as shown in Fig. 6. The other ends of these thermal couples were connected to the data collection chip. Moreover, the chip was linked to a PC to automatically collect and record the data using a professional software. Additionally, we used the heater made by nickel–chrome material to reduce its specific heat capacity.

#### 4.3. Measurement

Assuming that the thickness of the sample is the characteristic dimension, the Fourier number can be defined as

$$Fo = \alpha t / \delta^2, \quad (27)$$

Substituting Eq. (27) into Eq. (26), then it can be rewritten as

$$T = T_0 + \frac{2q\delta}{k} \sqrt{\frac{Fo}{\pi}} \left\{ \exp\left(-\frac{x^2}{4\delta^2 Fo}\right) + \exp\left[-\frac{(2\delta - x)^2}{4\delta^2 Fo}\right] \right\} - \frac{q}{k} \left[ x \operatorname{erf}\left(\frac{x}{2\delta\sqrt{Fo}}\right) + (2\delta - x) \operatorname{erf}\left(\frac{2\delta - x}{2\delta\sqrt{Fo}}\right) \right]. \quad (28)$$

In the experiment, we can conveniently measure the temperature at the top and bottom surfaces of the sample, which are  $T_1$  and  $T_2$ , respectively. When the temperature

$T_1$  has been measured, we can substitute  $x = 0$  into Eq. (28) and then obtain

$$T_1 = T_0 + \frac{2q\delta}{k} \left\{ \sqrt{\frac{Fo}{\pi}} \left[ 1 + \exp\left(-\frac{1}{Fo}\right) \right] - \operatorname{erf}\left(\frac{1}{\sqrt{Fo}}\right) \right\}. \quad (29)$$

Moreover, when the temperature  $T_2$  has been measured, we can substitute  $x = \delta$  into Eq. (28) and then achieve

$$T_2 = T_0 + \frac{2q\delta}{k} \left[ 2\sqrt{\frac{Fo}{\pi}} \exp\left(-\frac{1}{4Fo}\right) - \operatorname{erf}\left(\frac{1}{2\sqrt{Fo}}\right) \right]. \quad (30)$$

The parameters, including  $T_1$ ,  $T_2$ , and the voltage of the heater, were measured by the data collection chip. First, the signals of voltage deviation were collected by the chip, and then it was input to the PC. Secondly, a set of data of  $T_1$  and  $T_2$  were obtained via the software which can transform the voltage difference signals to the temperature signals. Finally, we can simultaneously calculate the thermal conductivity  $k$  and the thermal diffusivity  $\alpha$  based on Eqs. (29) and (30) using the measured  $T_1$  and  $T_2$ . We also can predict the specific heat  $c_p$  if the density  $\rho$  of the sample is known. In this paper, however, the objective is to investigate the thermal conductivity of wood, so the thermal diffusivity and specific heat of wood will not be discussed.

## 5. Analysis and comparison

According to the available literature data, the thermal conductivity  $k_w$  of the wood cell wall is 0.654 W/mK [2]. In addition, the wood samples we considered in the proposed fractal model are dry so that there is only air in the cavity of cells, whose thermal conductivity  $k_c$  is 0.0256 W/mK [2]. Then the thermal conductivities perpendicular to the fibres of these four kinds of wood, which are calculated using the proposed fractal model, were plotted in Fig. 7.

As displayed in Fig. 7, the tangential effective thermal conductivity is almost the same as the radial effective thermal conductivity. It is also obviously seen that the thermal conductivities almost linearly decrease with the increment

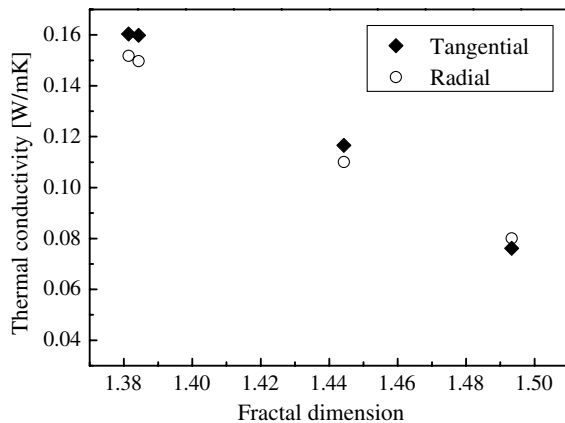


Fig. 7. Variation of thermal conductivities with fractal dimensions.

Table 3

Comparison of the thermal conductivities of wood in radial direction (W/mK)

Wood sample	Fractal model	Experimental data	Data reported in [13]
Larch	0.151	0.131	0.134
Korean pine	0.110	0.109	0.107

of the fractal dimensions. However, according to Eqs. (18) and (19), the thermal conductivities are not linear functions of the fractal dimension, so we could not directly explain this tendency using the fractal model, and it needs further investigation. Moreover, the tangential effective thermal conductivity is often a little bigger than the radial one. The accuracy is within 7–10% for both the tangential and the radial direction of the effective thermal conductivity under every factor taken into consideration. Due to the establishing process of the fractal model, there is no difference from these two directions. Therefore, the difference of the effective thermal conductivities of wood in these two directions owes to the macrostructure of wood, and it is cannot be represented by this fractal model.

The calculated results of larch and Korean pine are in good agreement with the experimental data as well as the available literature data as displayed in Table 3. The estimated uncertainty for measured value of thermal conductivity is within 3.2%. The comparison clearly shows that the prediction of effective thermal conductivities perpendicular to the fibres of wood via the proposed fractal model is credible, either in the tangential or in the radial direction. However, due to the complexity of the distribution of wood fibres, the effective thermal conductivities of different kinds of wood or perhaps different samples of the same tree are extremely different. Therefore, the calculated and experimental results can only be considered as the approximation of the effective thermal conductivities of wood. Moreover, the results of this fractal model only represent the effective thermal conductivities of the dry wood. Usually, for the reason that wood often contains some water, the model should be revised before using. Assuming that the moisture only exists in the cavity of cells, the thermal conductivity  $k_c$  of cell cavity is therefore greater than that of the air but lower than that of the water. The higher the moisture content, the greater the thermal conductivity  $k_c$  will be. Through the revision of the thermal conductivity  $k_c$ , the fractal model also can be used to predict the thermal conductivity of wet wood with different moisture contents.

## 6. Conclusions

In previous references, the relation between wood thermal properties and the porosity has studied, but the investigations of the porosity as well as thermal properties of wood using fractal theory are absent. In this paper, the relation between the fractal dimension and the porosity of wood has been established, then a fractal model is

proposed for predicting the effective thermal conductivities perpendicular to the fibres of wood. We also redesign the TPS method for measurement of the thermal conductivities of wood to validate the fractal model. Major conclusions are drawn as follow:

- (1) The fractal dimensions of different kinds of wood in their cross-section can be determined from the SEM images using the box-counting method.
- (2) The proposed fractal model can be used to conveniently predict both the tangential and the radial effective thermal conductivities of wood.
- (3) The improved TPS method can effectively measure the thermal properties of wood samples at a short time, and the results are extremely credible and precise.

In sum, we attempt to use the fractal theory to predict the effective thermal conductivity of wood in this paper. We succeed in proposing a fractal model for calculating the effective thermal conductivities perpendicular to the fibres of wood, but it is only a simple and rough model because it is based on the single cell simplification. Therefore, further work would be needed, especially for extending the fractal model to the three-dimensional condition.

#### Acknowledgements

We thank professor Chuan-Jing Tu at Zhejiang University for his helpful comments. This work was financially supported by the National Basic Research Program (973 Program) of China (No. 2001CB409600).

#### References

- [1] H. Thunman, B. Leckner, Thermal conductivity of wood—models for different stages of combustion, *Biomass Bioenerg.* 23 (2002) 47–54.
- [2] Y. Asako, H. Kamikoga, H. Nishimura, et al., Effective thermal conductivity of compressed woods, *Int. J. Heat Mass Transfer* 45 (2002) 2243–2253.
- [3] B.B. Mandelbrot, *The Fractal Geometry of Nature*, W.H. Freeman and Company, San Francisco, 1982.
- [4] R. Pitchumani, S.C. Yao, Correlation of thermal conductivities of unidirectional fibrous composites using local fractal techniques, *J. Heat Tran. ASME Trans.* 113 (1991) 788–796.
- [5] B. Yu, P. Cheng, Fractal models for the effective thermal conductivity of bidispersed porous media, *AIAA J. Thermophys. Heat Transfer* 16 (2002) 22–29.
- [6] B.X. Wang, L.P. Zhou, X.F. Peng, A fractal model for predicting the effective thermal conductivity of liquid with suspension of nanoparticles, *Int. J. Heat Mass Transfer* 46 (2003) 2665–2672.
- [7] S.E. Gustafsson, Transient plane source technique for thermal conductivity and thermal diffusivity measurements of solid materials, *Rev. Sci. Instrum.* 62 (1991) 797–804.
- [8] M. Gustavsson, E. Karawacki, S.E. Gustafsson, Thermal conductivity, thermal diffusivity, and specific heat of thin samples from transient measurement with hot disk sensors, *Rev. Sci. Instrum.* 65 (1994) 3856–3859.
- [9] B.M. Suleiman, J. Larfeldt, B. Leckner, et al., Thermal conductivity and diffusivity of wood, *Wood Sci. Technol.* 33 (1999) 465–473.
- [10] V. Bohac, M.K. Gustavsson, L. Kubicar, et al., Parameter estimations for measurement of thermal transport properties with the hot disk thermal constants analyzer, *Rev. Sci. Instrum.* 71 (2000) 2452–2455.
- [11] B. Fredlund, A model for heat and mass transfer in timber structures during fire, Lund University, Ph.D. Dissertation, 1988.
- [12] H.O. Peitgen, D. Saupe, M.F. Barnsley, et al., *The Science of Fractal Images*, Springer, New York, 1988.
- [13] J.Q. Cheng, *Wood Science* (in Chinese), Chinese Forest Press, Beijing, 1985.